p.h. sem-II paper-VI, unit-I
complex integration.

Proved

(1)

Complex variable.

Domain (Region) :->

A set of points in the Argand plane is said to be connected set if any two of its points can be joined by a continuous curve all of whose points belong to S.

Contour :->

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By Contour, we mean a continuous chain of a finite number of regular arcs.

If the contour is closed and does not intersect itself then it is called closed Contour.

Example Boundaries of Circle, Δ^s and rectangle.

Cauchy's theorem :-> (Remember)

If a function $f(z)$ is analytic and single valued inside and

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If $f(z)$ is an analytic function of z and if $f'(z)$ is continuous at each point within and on a closed contour C, then

$$\int_C f(z)\,dz = 0$$

where C is any closed contour contained in D

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			1	2	3	4
5	6	7	8	9	10	11
12	13	14	15	16	17	18
19	20	21	22	23	24	25
26	27	28	29	30		

(2)

simple closed contour C, then
$$\int_C f(z)\,dz = 0$$
where C is any closed contour contained in D.

Proof :→ In the proof of this theorem we will apply Green's theorem for a plane which states :

If $P(x,y)$, $Q(x,y)$, $\frac{\partial P}{\partial y}$, $\frac{\partial Q}{\partial x}$ are all continuous (without breaks) functions within a domain D and if C is any closed contour in D, then

$$\int_C (P\,dx + Q\,dy) = \iint_D \left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y}\right) dx\,dy$$

Now, $$\int_C f(z)\,dz = \int_C (u\,dx - v\,dy) + i\int_C (v\,dx + u\,dy) \quad \text{—— (1)}$$

we have
$$f'(z) = u_x + i v_x = v_y - i u_y \quad \text{—— (2)}$$
[By Cauchy's-Riemann Eq's]

Here $f'(z)$ is continuously differentiable so from (2) u_x, u_y, v_x, v_y all exist and are continuous in D.

Thus from Green's theorem

from (1)

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$$\int_C f(z)\,dz = \iint_D \left(-\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right) dx\,dy$$

$$+ i \iint_D \left(\frac{\partial u}{\partial x} - \frac{\partial v}{\partial y}\right) dx\,dy$$

$$= \iint_D \left(-\cancel{\frac{\partial v}{\partial x}} + \cancel{\frac{\partial v}{\partial x}}\right) dx\,dy$$

$$+ i \iint_D \left(\cancel{\frac{\partial u}{\partial x}} - \cancel{\frac{\partial u}{\partial x}}\right) dx\,dy$$

[By Cauchy's Reimann equations]

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$= 0$ Proved